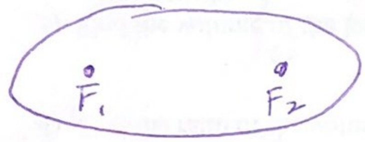
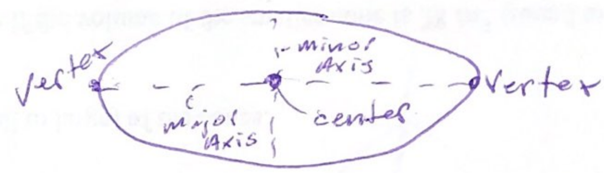
Chapter 9.1 The Ellipse

- look up ellipse video.



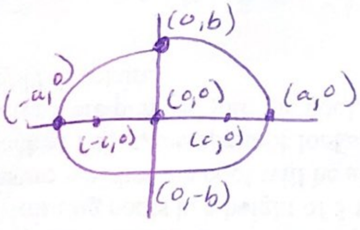


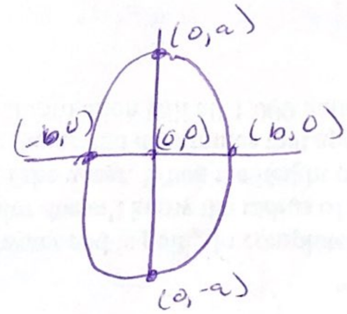
Foci 1 & 2

Standard Forms of the Equations of an Ellipse.

important

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{or} \quad \frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$





- 'a' goes under major axis

foci: $c^2=a^2-b^2$

① Graph ellipses centered at the origin, major axis

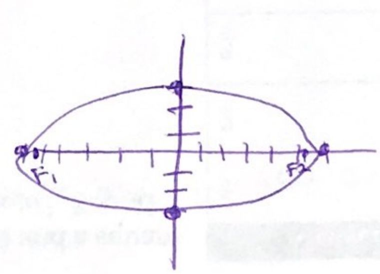
Ex] Graph and locate the foci: $\frac{x^2}{36}+\frac{y^2}{9}=1$

$a^2=36 \quad b^2=9$

$a=6 \quad b=3$

$c^2=6^2-3^2=36-9=27$

$c=\pm\sqrt{27}\approx\pm 5.19$



1] Graph and locate the foci: $16x^2+9y^2=144$

$$\frac{16x^2}{144}+\frac{9y^2}{144}=\frac{144}{144}$$

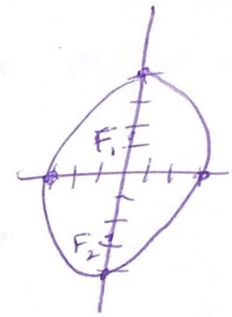
$$\frac{x^2}{9}+\frac{y^2}{16}=1$$ major axis

$a^2=16 \quad b^2=9$

$a=4 \quad b=3$

$c^2=a^2-b^2=16-9=7$

$c=\pm\sqrt{7}\approx\pm 2.65$



(2) Write equations of ellipses in standard form.

Ex] Find the standard form of the equation of an ellipse with foci at $(-2,0)$ and $(2,0)$ and vertices $(-3,0)$ and $(3,0)$

x is major axis

$\therefore c=2$

$a=3$

$c^2=a^2-b^2$

$b^2=a^2-c^2$

$b^2=3^2-2^2$

$b^2=9-4$

$\sqrt{b^2}=\sqrt{5}$ stop

$b=\pm\sqrt{5}\approx\pm 2.24$

$$\frac{x^2}{9}+\frac{y^2}{5}=1$$

(3) Graph ellipses not centered at the origin.

Ex] Graph: $\dfrac{(x+1)^2}{9} + \dfrac{(y-2)^2}{4} = 1$. Where are the foci located?

This is $(x-h)^2$ ↑ $(y-k)^2$

center is at $(-1, 2)$

$a^2 = 9$ $\qquad$ $b^2 = 4$ $\qquad$ Major axis is x-axis

$a = 3$ $\qquad$ $b = 2$

find c

$c^2 = a^2 - b^2$

$c^2 = 3^2 - 2^2$

$c^2 = 9 - 4$

$c^2 = 5$

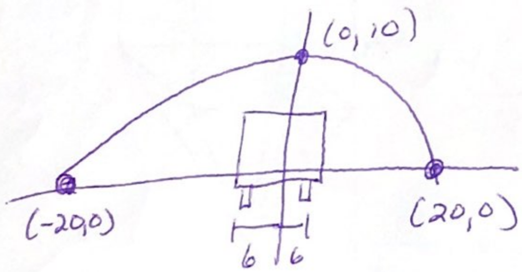
$c = \pm\sqrt{5}$

foci @ $(-1+\sqrt{5}, 2)$
$(-1-\sqrt{5}, 2)$

* Go to page 4 for next example

(4) Solve applied problems involving ellipses

Ex] Will a truck that is 12 feet wide and has a height of 9 ft clear the opening?



use the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{x^2}{400} + \frac{y^2}{100} = 1$$

Figure the height of ~~a~~ archway 6 ft from center

$$\frac{6^2}{400} + \frac{y^2}{100} = 1$$

$$\frac{36}{400} + \frac{y^2}{100} = 1$$

$$400\left(\frac{36}{400} + \frac{y^2}{100}\right) = 1 \cdot 400$$

$$36 + 4y^2 = 400$$

$$4y^2 = 364$$

$$\sqrt{y^2} = \sqrt{91}$$

$$y = 9.54$$

Yes the truck will fit.

5) convert $9x^2+4y^2-18x+16y-11=0$ to standard form

$$9x^2 - 18x + 4y^2 + 16y = 11$$

$$9(x^2-2x+\underline{1}) + 4(y^2+4y+\underline{4}) = 11 + \underline{9} + \underline{16}$$

$\frac{-2}{2} = -1 \Rightarrow (-1)^2 = 1$ $\quad$ $\frac{4}{2} = 2 \Rightarrow 2^2 = 4$

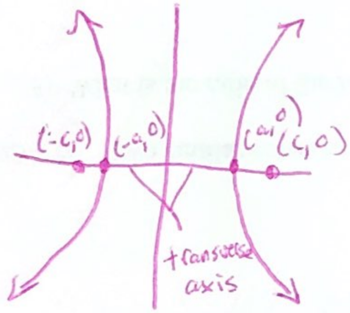
$$\frac{9(x-1)^2}{36} + \frac{4(y+2)^2}{36} = \frac{36}{36}$$

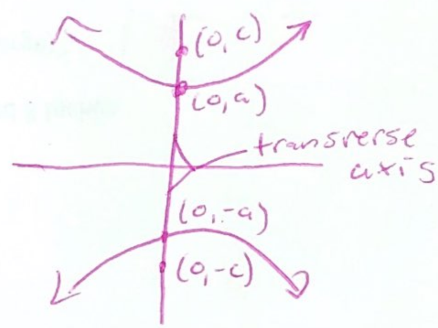
$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

Chapter 9.2 Hyperbola

Standard forms of the Equations of a Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$





① Locate a hyperbola's vertices and foci.

Ex | Find vertices ; foci

a.) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

$a^2 = 25$

$a = 5$

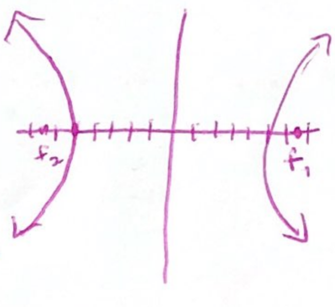
$c^2 = a^2 + b^2$

$c^2 = 25 + 16$

$\sqrt{c^2} = \sqrt{41}$

$c = \pm\sqrt{41} \approx 6.4$

$(\sqrt{41}, 0)$; $(-\sqrt{41}, 0)$



b.) $\frac{y^2}{25} - \frac{x^2}{16} = 1$

$a^2 = 25$

$a = 5$

$c^2 = a^2 + b^2 = 25 + 16 = 41$

$\sqrt{c^2} = \sqrt{41}$

$c = \pm\sqrt{41}$

Vertices : $(0, 5)$; $(0, -5)$

foci : $(0, \sqrt{41})$; $(0, \sqrt{41})$

Write equations of hyperbolas in standard form. C9.2

Ex | Find the standard form of the eq. of a hyperbola with foci at (0, -5) and (0, 5) and vertices (0, -3) & (0, 3)

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$a = 3$

$a^2 = 9$

$c = 5$

$c^2 = 25$

$c^2 = a^2 + b^2$

$25 = 9 + b^2$

$16 = b^2$

$$\boxed{\frac{y^2}{9} - \frac{x^2}{16} = 1}$$

→ The Asymptotes of a Hyperbola

If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$

If $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ then $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$

③ Graph hyperbolas centered at the origin.

Ex | Graph and locate the foci: $\frac{x^2}{36} - \frac{y^2}{9} = 1$

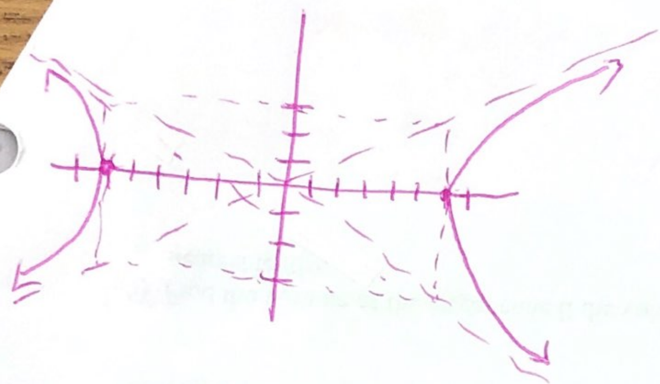
what are the eqs of asymptote.

Step 1: locate the vertices

$a^2 = 36$ $\qquad$ $b^2 = 9$

x-axis $a = \pm 6$ ← vertex $\qquad$ $b = \pm 3$

Step 2: Draw a rectangle.



Step 3: Draw extended diagonals for the rectangle to obtain the asymptotes

$y = \pm\frac{b}{a}x = \pm\frac{3}{6}x = \boxed{\pm\frac{1}{2}x}$

Step 4: Draw the 2 branches of the hyperbola

$c^2 = a^2 + b^2$

$c^2 = 36 + 9 = 45$

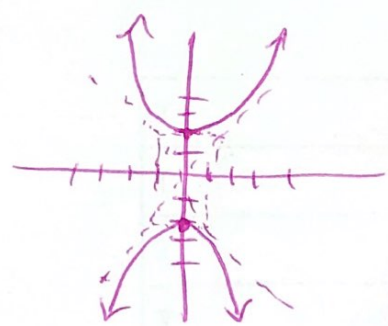
$c = \pm\sqrt{45}$

The foci are at $(\sqrt{45}, 0)$ & $(-\sqrt{45}, 0)$ or $(3\sqrt{5}, 0)$ & $(-3\sqrt{5}, 0)$

Ex] Graph & locate foci: $y^2 - 4x^2 = 4$, Asymptotes

$\frac{y^2 - 4x^2}{4} = \frac{4}{4}$

vertex → $\frac{y^2}{4} - \frac{x^2}{1} = 1$



$a^2 = 4$ $b^2 = 1$

$\boxed{a = \pm 2}$ $b = \pm 1$

→ $y = \pm\frac{a}{b}x = \pm\frac{2}{1}x = \boxed{\pm 2x}$

→ foci $c^2 = a^2 + b^2$

$c^2 = 4 + 1 = 5$

$c = \pm\sqrt{5}$ $(0, \sqrt{5})$ & $(0, -\sqrt{5})$

1) Graph hyperbolas not centered at the origin $\boxed{C\,9.2}$

$$(x-h)^2 \qquad (x-k)^2$$

[Ex] Graph : $\dfrac{(x-3)^2}{4} - \dfrac{(y-1)^2}{1} = 1$ foci & Asymptotes?

Center : $(3,1)$

$a^2 = 4$

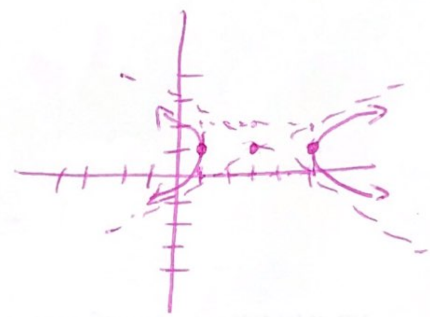
$\boxed{a = \pm 2}$

$b^2 = 1$

$b = \pm 1$

Asymptote

$y = \pm \frac{b}{a} x = \pm \frac{1}{2} x$



eq. in h/k form

$\boxed{y - 1 = \frac{1}{2}(x-3)}$

foci

$c^2 = a^2 + b^2$

$c^2 = 4 + 1$

$\sqrt{c^2} = \sqrt{5}$

$c = \pm\sqrt{5}$

$\boxed{(3+\sqrt{5}, 1) \,\&\, (3-\sqrt{5}, 1)}$

$\overline{C9.2}$

Graph: $4x^2-24x-9y^2-90y-153=0$. foci & Asymptote?

$(4x^2-24x)+(-9y^2-90y)=153$

$4(x^2-6x+\underline{9})-9(y^2+10y+\underline{25})=153+\underline{36}+\underline{-225}$

$-\frac{6}{2}=-3 \Rightarrow (-3)^2=9 \qquad \frac{10}{2}=5 \Rightarrow 5^2=25$

$$\frac{4(x-3)^2}{-36}-\frac{9(y+5)^2}{-36}=\frac{-36}{-36}$$

$$-\frac{(x-3)^2}{9}+\frac{(y+5)^2}{4}=1$$

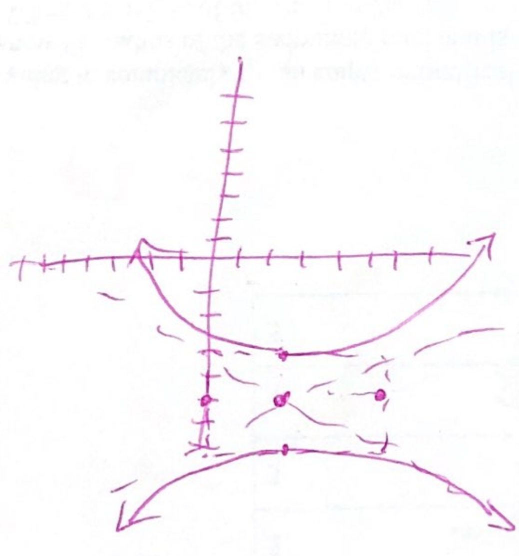
$$\frac{(y+5)^2}{4}-\frac{(x-3)^2}{9}=1$$

Center: $(3,-5)$

y-axis $a^2=4 \qquad b^2=9$

$a=\pm 2 \qquad b=\pm 3$

Vertices: $(3,-3)$ & $(3,-7)$



Asymptotes

$y=\pm\frac{a}{b}x=\pm\frac{2}{3}x$

$(y+5)=\pm 2/3(x-3)$

foci:

$c^2=a^2+b^2$

$c^2=4+9$

$\sqrt{c^2}=\sqrt{13}$

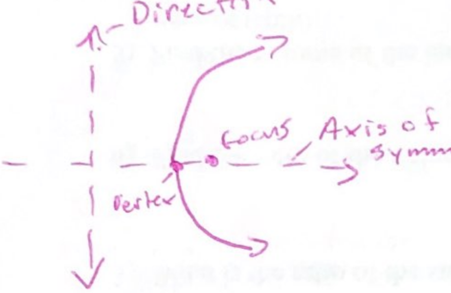
$(3,-5-\sqrt{13})$ & $(3,-5+\sqrt{13})$

$\overline{Pg\ 5}$

Chapter 9.3 The Parabola

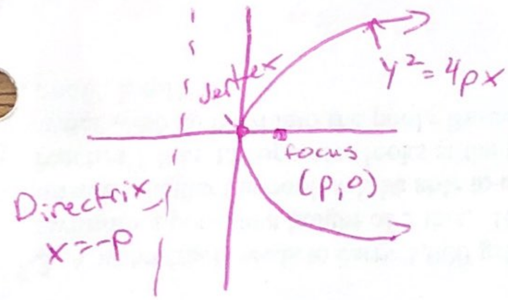
We know $y = a(x-h)^2 + k$ or $y = ax^2 + bx + c$

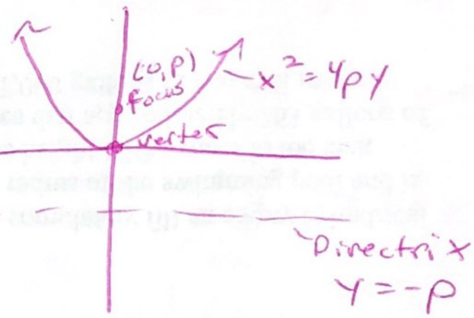
but now!



Standard form of the Equation of a Parabola

$y^2 = 4px$ or $x^2 = 4py$



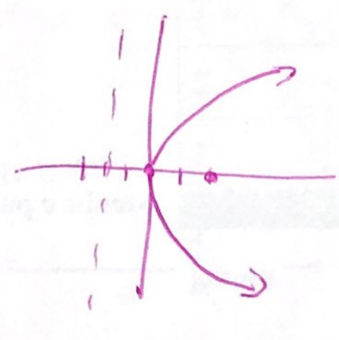


① Graph parabolas centered @ origin

Ex) Find the focus & directrix of the parabola given by $y^2 = 8x$. Then graph.

$8 = 4p$ Directrix Focus

$2 = p$ $x = -2$ $(2, 0)$



he latus Rectum

C 9.3

- Found by $|4p|$ alligned w/ the focus

Ex | Focus, directrix and graph $x^2 = -12y$

$4p = -12$

$p = -3$

Focus

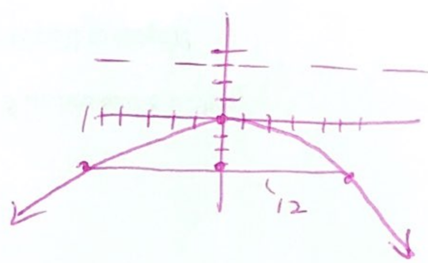
$(0, -3)$

Directrix

$y = -(-3) = 3$

latus Rectum

$|4p| = 12$



(2) Write eq. of parabolas in standard form

Ex | Standard form of parabola w/ focus $(8, 0)$ and directrix $x = -8$

$y^2 = 4px$

$p = 8$

$4p = 32$

$\boxed{y^2 = 32x}$

(3) Graph parabolas w/ vertices not on origin

Ex | Find vertex, focus, directrix and graph $(x-2)^2 = 4(y+1)$

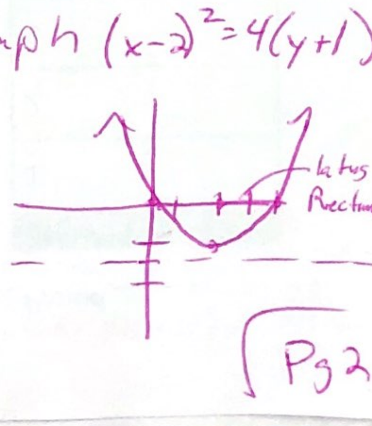
$4p = 4$

$p = 1$

Vertex: $(2, -1)$

Focus: $(h, k+p) = (2, -1+1) = (2, 0)$

directrix: $y = k - p = -1 - 1 = -2$



Pg 2

Ex) Find vertex, focus, directrix and graph. $\boxed{C\ 9.3}$

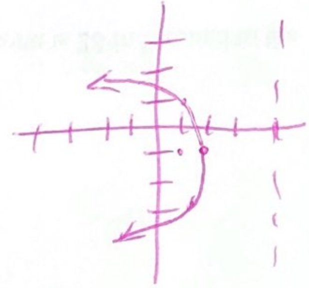
$y^2+2y+4x-7=0$

$y^2+2y+\underline{1}=-4x+7+\underline{1}$

$(y+1)^2=-4(x-2)$

$4p=-4$

$p=-1$



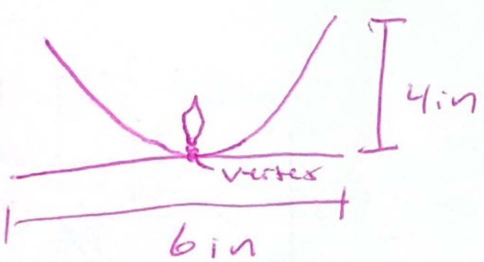
vertex: $(2,-1)$

focus: $(h+p,k)=(2+-1,-1)=(1,-1)$

directrix: $x=h-p=2-(-1)=3$

(4) Solve applied problems involving parabolas

Ex) We are designing a flashlight. The casting has a diameter of 6 in. and a depth of 4 in. What eq of the parabola used to shape the mirror? At what pt should the light source be placed relating to the mirror's vertex?



$x^2=4py$ focus $(0,p)$

let $x=3$ & $y=4$ (point on parabola)

$3^2=4p\cdot4$

$9=16p$

$p=\frac{9}{16}$

$x^2=4\left(\frac{9}{16}\right)y$

$x^2=\frac{9}{4}y$

The light should be placed at $\left(0,\frac{9}{16}\right)$ or $\frac{9}{16}$ in above vertex

9.5 Parametric Equations

*Watch video on Khan Academy

http://www.khanacademy.org/math/trigonometry/parametric_equations/parametric/v/parametric-equations-1

Plane Curves and Parametric Equations:

Plane curve is the set of ordered pairs (x, y) where

$x = f(t)$, $y = g(t)$ for t in interval I.

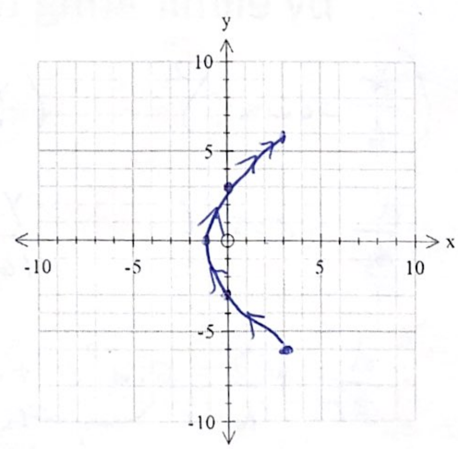
The variable t is called a parameter, and the equations $x = f(t)$ and $y = g(t)$ are called parametric equations for the curve.

Plane curve: The collection of points (x, y)

1) Graph the plane curve defined by the parametric equations:

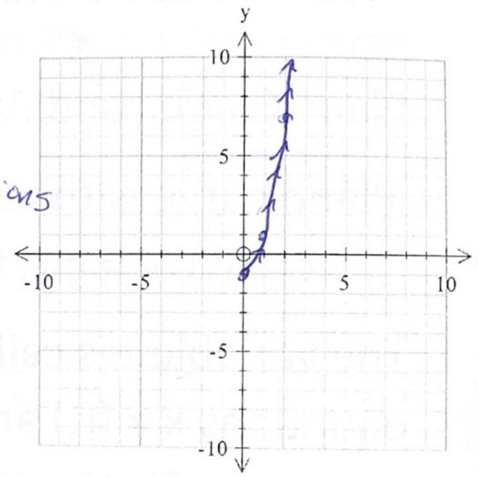
$x = t^2 - 1, \quad y = 3t \qquad -2 \le t \le 2$

t	t^2-1	$3t$
-2	3	-6
-1	0	-3
0	-1	0
1	0	3
2	3	6



Arrow follows time

2) Sketch the plane curve represented by the parametric equations x = $\sqrt{t}$ and y = 2t – 1 by eliminating the parameter.



* writing one equation in x & y that is equivalent to the two parametric equations

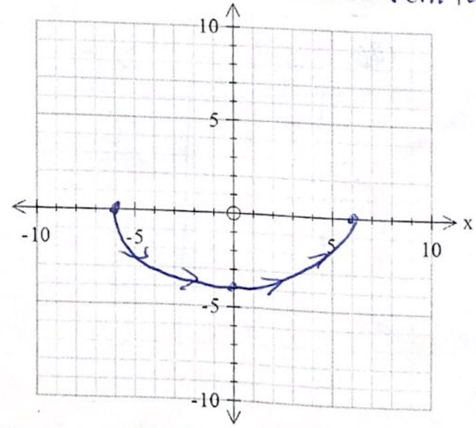
$x^2 = \sqrt{t}^{\,2}$

$t = x^2$ → $y = 2(x^2) - 1$

$y = 2x^2 - 1$

* $t \geq 0$

3) Sketch the plane curve represented by the parametric equations x = 6cos t, y = 4sin t, π≤t≤2π by eliminating the parameter. * use arrows to show orientation



$\left(\frac{x}{6} = \cos t\right)^2$ $\left(\frac{y}{4} = \sin t\right)^2$

$\frac{x^2}{36} = \cos^2 t$ $\frac{y^2}{16} = \sin^2 t$

$\frac{x^2}{36} + \frac{y^2}{16} = \underbrace{\sin^2 t + \cos^2 t}_{1}$

$\frac{x^2}{36} + \frac{y^2}{16} = 1$

t	x = cos	y = sin
π	−6	0
$\frac{3\pi}{2}$	0	−4
2π	6	0

Find the set of parametric equations for the parabola:

$x = t$; $y = f(t)$ t is the domain for f

4) $y = 9 - x^2$

☆ x can't have

Let $x = t$

$y = 9 - t^2$

$\boxed{x = t \; ; \; y = 9 - t^2}$

* The substitution for x must be a function that allows x to take on all the values in the domain of the given rectangular equation

5) $y = x^2 - 25$

$$\boxed{\begin{array}{l} x = t \\ y = t^2 - 25 \end{array}}$$

What unit am I doing?